POLYNOMIAL THEOREMS

Vieta's Formulas

Symmetric relationships between roots and coefficients.

THEOREM

For a monic polynomial of degree n, the k-th elementary symmetric polynomial of the roots equals $(-1)^k$ times the coefficient of x^{n-k}.

1. **QUADRATIC**
   $x^2 + bx + c$ with roots r_1, r_2: $r_1 + r_2 = -b$ and $r_1 r_2 = c$.

2. **CUBIC**
   $x^3 + bx^2 + cx + d$: $r_1 + r_2 + r_3 = -b$, $r_1 r_2 + r_1 r_3 + r_2 r_3 = c$, $r_1 r_2 r_3 = -d$.

3. **CHECK A FACTORIZATION**
   $x^3 - 6x^2 + 11x - 6$ with roots $1, 2, 3$: $1 + 2 + 3 = 6 = -(-6)$, $1 \cdot 2 + 1 \cdot 3 + 2 \cdot 3 = 11$, $1 \cdot 2 \cdot 3 = 6 = -(-6)$.

4. **WITHOUT THE ROOTS**
   Sum of the squares of the roots: $(r_1 + r_2 + r_3)^2 - 2(r_1 r_2 + r_1 r_3 + r_2 r_3) = 36 - 22 = 14$.

TAKING THE SUM OF THE ROOTS AS b

Reading the sum of the roots of $x^2 - 5x + 6$ as $b = -5$ clashes with its roots 2 and 3. The sum is $-b = 5$ and the product is $c = 6$.