NOTATION · CHEAT SHEET

Vector Space Notation at a Glance

The marks from the notation cards across the vector-space pages: how each is read and what it means. Every group links to the full card on its page.

AXIOMS

$0\mathbf{v} = \mathbf{0}$	**ZERO TIMES V IS THE ZERO VECTOR** Plain 0 is a number, bold $\mathbf{0}$ a vector.
$c(d\mathbf{v}) = (cd)\mathbf{v}$	**TWO KINDS OF PRODUCT** cd multiplies numbers; $d\mathbf{v}$ scales a vector.
$\mathbf{u} \oplus \mathbf{v}$	**U CIRCLE-PLUS V** An addition defined for the occasion, not the usual one.

SUBSPACES

$W \subseteq V$	**W IS CONTAINED IN V** Needed for a subspace, but not enough.
$\{\mathbf{0}\}$ and V	**THE TRIVIAL SUBSPACES** Every space has these two.
$W_1 \oplus W_2$	**DIRECT SUM** A sum with $W_1 \cap W_2 = \{\mathbf{0}\}$.

SPAN

$\mathrm{Span}\{\mathbf{v}_1, \ldots\}$	**THE SPAN OF THE VECTORS** All their linear combinations.
$\langle \mathbf{v}_1, \ldots \rangle$	**THE SUBSPACE GENERATED BY** The algebraist's spelling of span.
$c_i \in \mathbb{F}$	**SCALARS FROM THE FIELD F** $\mathbb{F}$ stands for $\mathbb{R}$ or $\mathbb{C}$.

LINEAR INDEPENDENCE

$\sum c_i \mathbf{v}_i = \mathbf{0}$	**A VANISHING COMBINATION** Independent when this forces every $c_i = 0$.
some $c_i \neq 0$	**A DEPENDENCE RELATION** One combination that proves dependence.
$W(f_1, \ldots, f_n)(x)$	**THE WRONSKIAN** Tests independence of functions.

BASIS

$\mathcal{B}$ vs B	**SCRIPT B, ROMAN B** The set of basis vectors against the matrix built from them.
$[\mathbf{v}]_{\mathcal{B}}$	**COORDINATES OF V IN B** The column of weights on the basis vectors.
$P_{\mathcal{C} \leftarrow \mathcal{B}}$	**CHANGE OF BASIS, B TO C** Read right to left: from $\mathcal{B}$ to $\mathcal{C}$.

DIMENSION

$\dim(V)$	**THE DIMENSION OF V** The number of vectors in any basis.
$\dim(V) = \infty$	**INFINITE-DIMENSIONAL** No finite basis exists.
$V \cong W$	**V IS ISOMORPHIC TO W** Same structure, possibly different objects.

FUNDAMENTAL SPACES

$\mathrm{Col}, \mathrm{Row}, \mathrm{Null}$	**COLUMN, ROW AND NULL SPACE** Names vary by author, for example $C(A)$, $N(A)$.
$\mathrm{Null}(A^T)$	**THE LEFT NULL SPACE** Always written through the transpose.
$\mathbf{x} \mapsto A\mathbf{x}$	**X MAPS TO A X** The barred arrow acts on single vectors.

SAME MARKS, DIFFERENT JOB

$\oplus$	A made-up addition in the axioms; a direct sum of subspaces.
$\langle \cdot \rangle$	Generated subspace here; inner product on the orthogonality pages.
$W(\ldots)$	The Wronskian, not a subspace named W.