TWO LINES IN THE PLANE

Each equation of a linear system is a hyperplane, and the solution set is where they all meet. For two equations in two unknowns each equation is a line in $\mathbb{R}^2$, and how the two lines lie decides the outcome.

TWO EQUATIONS, TWO UNKNOWNS

two lines in $\mathbb{R}^2$

split by **how the two lines lie**

1

CASE 1

distinct, not parallel

Meet at one point

solution

a unique solution

0

CASE 2

parallel

Never meet

solutions

no solution

∞

CASE 3

coincident

Overlap entirely

solutions

the same line described twice: infinitely many solutions

TAKEAWAY

The same classification persists in higher dimensions: three planes in $\mathbb{R}^3$ can meet in a point, along a line, or not at all, including the triangular prism arrangement where each pair meets but no point lies on all three.