

Subspaces built from two subspaces

From two subspaces W_1 and W_2 of V , the intersection and the sum are again subspaces, and they bracket each of the originals.

$$W_1 \cap W_2 \subseteq W_1 \subseteq W_1 + W_2 \subseteq V$$

V

The whole space

$W_1 + W_2$ **Sum**

adds **all combinations** · the smallest subspace containing both

W_1

Subspace

one of the two originals

$W_1 \cap W_2$ **Intersection**

always a subspace; between $\{0\}$ and one of the originals

Dimension bookkeeping

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$. The union, unlike these, is almost never a subspace.

