

Three moves, all reversible

Gaussian elimination uses only three operations on rows. Each one changes the system without changing its solutions, and each can be undone.

Elementary row operations

3 operations

Swap

$$R_i \leftrightarrow R_j$$

Does
reorders equations, fixes a zero pivot

Undone by
the same swap

Scaling

$$kR_i, k \neq 0$$

Undone by
scaling by $1/k$

$k = 0$ is forbidden

Addition

$$R_i + cR_j$$

Does
eliminates entries

Undone by
subtracting cR_j

Only one move eliminates

Swapping and scaling tidy the rows; adding a multiple of one row to another is the operation that actually creates the zeros.

