ODD OR EVEN N

The n-th roots of unity always sum to zero. Their product and which of them are real depend on whether n is odd or even.

ROOTS OF UNITY

$$\prod_{k=0}^{n-1} z_k = (-1)^{n+1}$$

split by **the parity of n**

1

CASE 1

n odd

Product is 1

product of all roots

only $z = 1$ sits on the real axis

−1

CASE 2

n even

Product is −1

product of all roots

$z = 1$ and $z = -1$ sit on the real axis

WHY

The product is the constant term of $z^n - 1 = (z - z_0)(z - z_1) \cdots (z - z_{n-1})$, which gives $(-1)^{n+1}$. Every other root has its conjugate among the roots.