REAL CANONICAL FORM

A rotation-scaling block

Instead of a diagonal matrix with complex entries, a real 2 × 2 block: a rotation by θ scaled by r.

BEFORE

Standard basis

$$A = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$$

Eigenvalues 2 ± i

CHANGE BASIS

P = [w u]: w = (1, 0), then u = (1, 1)

IMAGINARY PART FIRST

AFTER

Basis w, u

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

a = 2, b = 1, so r = √5, θ ≈ 26.6°

WHAT CHANGED	The basis: in the basis w, u the map reads as a rotation by θ ≈ 26.6° scaled by r = √5.
WHAT STAYED	The map and its eigenvalues 2 ± i; every entry stays real.