POLYNOMIAL THEOREMS

The Rational Root Theorem

An infinite search becomes a finite one.

THEOREM

If $P(x)$ has integer coefficients and $\frac{p}{q}$ is a rational root in lowest terms, then $p \mid a_0$ and $q \mid a_n$.

1 READ THE ENDS

$2x^3 + x^2 - 13x + 6$: constant term 6, leading coefficient 2.

2 DIVISORS

p: $\pm 1, \pm 2, \pm 3, \pm 6$ · q: $\pm 1, \pm 2$

3 CANDIDATES

$\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$: twelve in all.

4 TEST

$P(1) = -4$, not a root. $P(2) = 16 + 4 - 26 + 6 = 0$, so 2 is a root and $(x - 2)$ is a factor.

5 REDUCE

Synthetic division leaves $2x^2 + 5x - 3 = (2x - 1)(x + 3)$: roots 2, $\frac{1}{2}$, -3, all on the list.

LEAVING OUT THE FRACTIONS

Listing only the divisors of 6 gives $\pm 1, \pm 2, \pm 3, \pm 6$ and misses the root $\frac{1}{2}$. Dividing by $q = \pm 2$ adds $\pm \frac{1}{2}$ and $\pm \frac{3}{2}$.