POLYNOMIAL THEOREMS

Limits of the Rational Root Theorem

It identifies only rational roots.

THE THEOREM SAYS

If $\frac{p}{q}$ is a rational root of a polynomial with integer coefficients, then p divides a_0 and q divides a_n.

IT DOES NOT SAY

~~Every polynomial has a rational root, or every candidate on the list is a root.~~

READING AN EMPTY LIST AS "NO ROOTS"

WRONG WORKING

$x^2 - 2$ has candidates $\pm 1, \pm 2$; none is a root, so the polynomial has no roots.

RIGHT WORKING

The roots are $\pm\sqrt{2}$, irrational and invisible to the theorem. $x^4 + x^3 + x^2 + x + 1$ has candidates ± 1 and four complex roots.

A first filter, not a complete solution: when it finds roots, synthetic division lowers the degree; when it finds none, the roots are irrational or complex.