HOW MUCH DIMENSIONALITY IS USED

The rank of an $m \times n$ matrix is a single non-negative integer that can never exceed $\min(m, n)$. Whether it reaches that ceiling splits matrices into two conditions.

RANK BOUNDS

$$0 \leq \text{rank}(A) \leq \min(m, n)$$

split by $\text{rank}(A)$ compared with $\min(m, n)$

CASE 1

$$\text{rank}(A) = \min(m, n)$$

Full rank

every row and every column contributes something that no combination of the others can reproduce

E.G. a 5×3 matrix with rank 3 uses all three column directions

CASE 2

$$\text{rank}(A) < \min(m, n)$$

Rank-deficient

at least one row or column is a linear combination of the others

E.G. a 5×3 matrix with rank 2 has one redundant column, in the span of the other two

TAKEAWAY

The rank does not say which row or column is redundant, often more than one subset works; it only gives the effective count.