BEFORE YOU READ

Each condition narrows the set

A set of vectors that is orthonormal is automatically orthogonal, and an orthogonal set of nonzero vectors is automatically linearly independent.

$$\text{ON} \subset \perp \subset \text{LI}$$

LI — Linearly independent sets

no vector is a combination of the others

$\perp$ — Orthogonal sets

nonzero vectors, pairwise $\mathbf{v}_i \cdot \mathbf{v}_j = 0$

ON — Orthonormal sets

orthogonal, and every vector has unit length

TAKEAWAY

Why orthonormal bases are prized

In an orthonormal basis the coordinates of a vector are just dot products with the basis vectors. See orthogonal sets.