NOTATION · CHEAT SHEET

Orthogonality Notation at a Glance

The marks from the notation cards across the orthogonality pages: how each is read and what it means. Every group links to the full card on its page.

INNER PRODUCT

$\langle \mathbf{u}, \mathbf{v} \rangle$	**THE INNER PRODUCT OF U AND V** Two vectors in, one number out.
$\langle \cdot, \cdot \rangle$	**THE INNER PRODUCT AS A FUNCTION** The dots are empty slots.
$\mathbf{u} \perp \mathbf{v},\ W^{\perp}$	**U ORTHOGONAL TO V; W PERP** $\langle \mathbf{u}, \mathbf{v} \rangle = 0$; $W^{\perp}$ holds all vectors orthogonal to W.

ORTHOGONAL SETS

δ_{ij}	**THE KRONECKER DELTA** 1 when $i = j$, 0 otherwise.
$\hat{\mathbf{u}} = \mathbf{v}/\lVert\mathbf{v}\rVert$	**U HAT** The unit vector in the direction of $\mathbf{v}$.
$Q^T Q = I$	**Q IS ORTHOGONAL** So $Q^{-1} = Q^T$.

PROJECTIONS

$\text{proj}_W \mathbf{b}$	**PROJECTION OF B ONTO W** The subscript is the target.
$\hat{\mathbf{b}}$	**B HAT** The closest point to $\mathbf{b}$ in W.
$P^2 = P$	**THE PROJECTION MATRIX** Projecting twice changes nothing.

GRAM–SCHMIDT

$\mathbf{v}_j \to \mathbf{u}_j \to \mathbf{e}_j$	**INPUT, ORTHOGONALISED, NORMALISED** Three letters for three stages of one vector.
$\sum_{i=1}^{j-1} \text{proj}_{\mathbf{u}_i}$	**ALL PROJECTIONS SO FAR** Subtracted from $\mathbf{v}_j$; the sum grows with j.
$A = QR$	**Q R FACTORISATION** Orthonormal Q, upper-triangular R.

LEAST SQUARES

$\hat{\mathbf{x}}$	**X HAT** The best fit: minimises $\lVert A\mathbf{x} - \mathbf{b} \rVert$.
$A^T A\hat{\mathbf{x}} = A^T \mathbf{b}$	**THE NORMAL EQUATIONS** Normal means perpendicular: the residual is orthogonal to the columns.
A^{+}	**A PLUS, THE PSEUDOINVERSE** Stands in for an inverse that does not exist: $\hat{\mathbf{x}} = A^{+}\mathbf{b}$.

SAME MARKS, DIFFERENT JOB

the hat	$\hat{\mathbf{u}}$ is a unit vector; $\hat{\mathbf{b}}$ and $\hat{\mathbf{x}}$ are best approximations.
P	Projection here; elsewhere, change of basis or a permutation.
$\perp$	Between vectors, orthogonal; as a superscript, the orthogonal complement.