PROPERTIES OF OPERATIONS

Which operation has which property?

Addition and multiplication share every property; subtraction and division lose the first two.

✓ HOLDS · ◐ CONDITIONAL · ✕ FAILS

PROPERTY	$+$	$\times$	$-$	$\div$
Commutative order does not matter	✓ HOLDS $a + b = b + a$	✓ HOLDS $a \cdot b = b \cdot a$	✕ FAILS $7 - 4 \neq 4 - 7$	✕ FAILS $7 \div 4 \neq 4 \div 7$
Associative grouping does not matter	✓ HOLDS any grouping	✓ HOLDS any grouping	✕ FAILS grouping matters	✕ FAILS grouping matters
Identity element leaves every input unchanged	✓ HOLDS $a + 0 = a$	✓ HOLDS $a \cdot 1 = a$	NOT ON THIS PAGE	NOT ON THIS PAGE
Inverse element undoes the operation	✓ HOLDS $a + (-a) = 0$	◐ CONDITIONAL $\frac{1}{a}$, for $a \neq 0$	NOT ON THIS PAGE	NOT ON THIS PAGE

Subtraction and division are derived operations: $a - b = a + (-b)$ and $\frac{a}{b} = a \cdot \frac{1}{b}$. **Distributive** links the two main ones: $a(b + c) = ab + ac$.