

① ONE DIRECTION AT A TIME

When A is diagonalizable, $\mathbf{x}' = A\mathbf{x}$ decouples into independent equations $y_i' = \lambda_i y_i$ with solutions $c_i e^{\lambda_i t}$. Each eigenvalue decides the behavior along its own eigenvector.

ALONG EIGENVECTOR $\mathbf{v}_i$

$$c_i e^{\lambda_i t} \mathbf{v}_i$$

split by the eigenvalue λ_i



CASE 1

$$\lambda_i > 0$$

Exponential growth



CASE 2

$$\lambda_i < 0$$

Decay



CASE 3

$$\lambda_i = 0$$

Constant component



CASE 4

λ_i complex

Oscillation

sines and cosines
modulated by exponential
envelopes; see [complex
eigenvalues](#)



TAKEAWAY

The constants $c_1, \dots, c_n$ come from the initial condition: write $\mathbf{x}(0)$ as a combination of the eigenvectors and read off the coefficients.