start

$\mathbb{N}$

Natural numbers

the counting numbers 1, 2, 3, ...

subtraction demands answers beyond the counting numbers

1st extension

$\mathbb{Z}$

Integers

the negatives join the counting numbers

division insists on exactness

2nd extension

$\mathbb{Q}$

Rational numbers

fractions: every division now has an exact result

+

geometry requires lengths no fraction can capture

3rd extension

$\mathbb{R}$

Real numbers

the irrationals fill the line

$x^2 + 1 = 0$ has no real solution

4th extension

$\mathbb{C}$

Complex numbers

a new unit i with $i^2 = -1$, perpendicular to the real line

EACH TIME declare the problem unsolvable, or expand what counts as a number. Mathematics chose to expand.

$\mathbb{N} \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{R} \to \mathbb{C}$

Select a level for details.