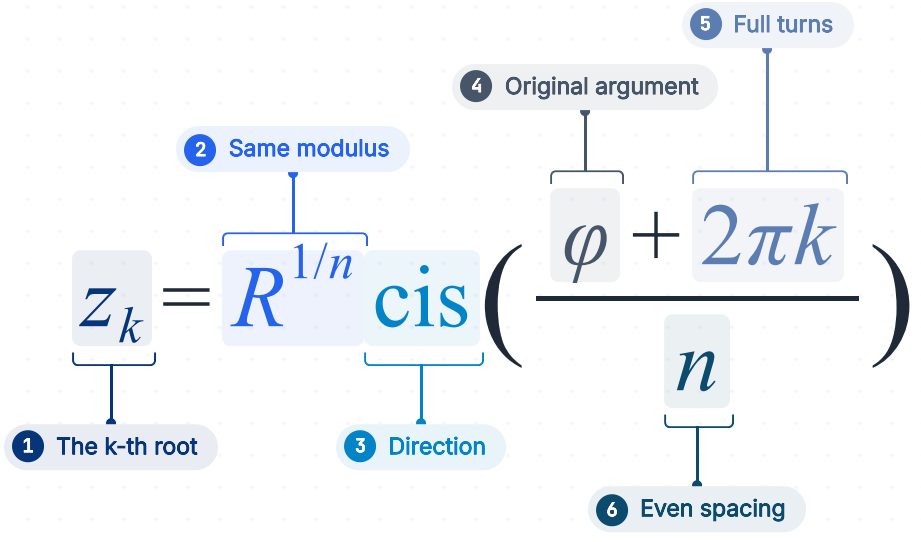
THE FORMULA, PART BY PART

The n-th roots of w = R cis φ

one modulus for all n roots; the arguments start at φ/n and step evenly around the circle



1 The k-th root z_k

Each value of k from 0 to n − 1 gives one root; together they are all solutions of z^n = w. Example: the cube roots of 8 are z_0 = 2, z_1 = −1 + √3 i and z_2 = −1 − √3 i. Each cubes to 8.

2 Same modulus $R^{1/n}$

The positive real n-th root of the original modulus, shared by all n roots; the roots differ only in their arguments. Example: 8 = 8 cis 0°, so R = 8 and every cube root has modulus $8^{1/3}$ = 2.

3 Direction cis

It turns the angle in brackets into a point on the unit circle; the modulus in front scales it out to radius $R^{1/n}$. Example: 2 cis 120° = 2(−1/2 + (√3/2) i) = −1 + √3 i.

4 Original argument φ

For k = 0 the argument is φ/n: the original angle divided by n. That first root is where the evenly spaced set starts. Example: φ = 0° for w = 8, so z_0 = 2 cis 0° = 2.

5 Full turns $k = 0 \ldots n-1$

Adding 2πk (360°k) leaves w unchanged, but after dividing by n each k lands somewhere new. Adding the turns after dividing gives the same root n times. Example: for z^3 = 8i, (90° + 360°k)/3 gives 30°, 150°, 270°.

6 Even spacing n

Dividing by n spreads the n roots evenly around the circle of radius $R^{1/n}$. Example: for the cube roots of 8 the step is 360°/3 = 120°, giving the angles 0°, 120° and 240°.