CONTRADICTIONS

Reducing to ⊥

The first conjunct requires Q to be false, the second requires Q to be true. The reduction makes the conflict explicit.

01 NEGATED IMPLICATION — BEFORE

$$\neg(P \to Q) \wedge (P \wedge Q)$$

↓ Negate the implication — P → Q ≡ ¬P ∨ Q, so ¬(P → Q) ≡ P ∧ ¬Q

02 TWO CONJUNCTIONS

$$(P \wedge \neg Q) \wedge (P \wedge Q)$$

↓ Regroup — P twice is just P; put ¬Q next to Q

03 Q AGAINST ¬Q

$$P \wedge (\neg Q \wedge Q)$$

↓ A statement and its negation — ¬Q ∧ Q is false outright: ⊥

04 CONJOINED WITH ⊥

$$P \wedge \bot$$

↓ Anything ∧ ⊥ collapses — P ∧ ⊥ = ⊥

05

FALSE OUTRIGHT

AFTER

$\bot$

NOT P → ¬Q