COMPUTING THE INVERSE

# Which method for the inverse?

Ask first whether the inverse itself is needed; most practical work stops there.

**EXISTS ONLY IF** $\det(A) \neq 0$. Every method below assumes an invertible matrix.

**1** Only the solution of $Ax = b$ needed?
not the matrix $A^{-1}$ itself

→ YES → **Solve $Ax = b$ directly**
Gaussian elimination, about $\frac{2}{3}n^3$ operations; the LU factors can be reused → When not to compute the inverse

NO ↓

**2** Is A a 2 × 2 matrix?

→ YES → **Swap-and-negate formula**
swap the diagonal entries, negate the off-diagonal ones, divide by ad − bc → The 2×2 formula

NO ↓

**3** Are the entries symbolic?
letters or parameters, not numbers

→ YES → **Adjugate formula**
$A^{-1} = \text{adj}(A) / \det(A)$: $n^2$ cofactors, each an $(n-1)$ determinant → Via the adjugate

→ NO → **Row reduction of $[A \mid I]$**
reduce to $[I \mid A^{-1}]$, about $2n^3$ operations; a singular A shows up during the reduction → By row reduction