INEQUALITIES

What each move does to the direction

Most moves keep the inequality as it is; a few flip it, and squaring cannot be trusted.

✓ KEEPS DIRECTION ◐ FLIPS DIRECTION ✕ UNRELIABLE

Add or subtract ✓ KEEPS DIRECTION

$$a < b \Rightarrow a + c < b + c$$

for any real c, exactly as with equations

Multiply or divide by a positive ✓ KEEPS DIRECTION

$$a < b,\ c > 0 \Rightarrow ac < bc$$

$c > 0$

Multiply or divide by a negative ◐ FLIPS DIRECTION

$$a < b,\ c < 0 \Rightarrow ac > bc$$

$c < 0$

the one rule that sets inequalities apart from equations

$3 < 5$, but multiplying by -1 gives $-3 > -5$: negation mirrors the number line.

Take reciprocals ◐ FLIPS DIRECTION

$$2 < 3 \Rightarrow \tfrac{1}{2} > \tfrac{1}{3}$$

both sides of the same sign

Transitivity ✓ KEEPS DIRECTION

$$a < b,\ b < c \Rightarrow a < c$$

why a chain like $1 < x < 7$ is allowed

Square both sides ✕ UNRELIABLE

$$-5 < 2 \text{ but } 25 > 4$$

squaring is not monotone: it can keep, flip or destroy the inequality

With opposite signs it does not flip: $-1 < 2$ and $-1 < \frac{1}{2}$.