POLYNOMIAL THEOREMS

The Fundamental Theorem of Algebra

Every polynomial of degree $n \geq 1$ has exactly n complex roots, counted with multiplicity.

THE THEOREM SAYS

A polynomial of degree $n \geq 1$ with complex coefficients has exactly n roots in the complex numbers, counted with multiplicity.

IT DOES NOT SAY

~~A polynomial of degree n has n real roots and crosses the x-axis n times.~~

EXPECTING ALL ROOTS TO BE REAL

WRONG WORKING

$x^3 - x^2 + x - 1$ has three roots, so its graph crosses the x-axis three times.

RIGHT WORKING

It factors as $(x-1)(x^2+1)$: one crossing at $x = 1$, the other two roots are $\pm i$. Repeats count too: $(x-1)^3$ has one distinct root with multiplicity 3.

Existence, not a method: for degree 5 and above no general formula in radicals exists (Abel–Ruffini). With real coefficients, complex roots come in conjugate pairs, so odd degree forces at least one real root.