

LONG-TERM BEHAVIOR

In dynamical systems, eigenvalues determine long-term behavior. For the discrete system $\mathbf{x}_{n+1} = A\mathbf{x}_n$ the deciding number is the absolute value of the eigenvalues.

DISCRETE DYNAMICAL SYSTEM

$$\mathbf{x}_{n+1} = A\mathbf{x}_n$$

split by $|\lambda|$ compared with 1



CASE 1

$$|\lambda| > 1$$

Grows



CASE 2

$$|\lambda| < 1$$

Decays



CASE 3

$$|\lambda| = 1$$

Oscillates



CONTINUOUS TIME

For $\mathbf{x}' = A\mathbf{x}$ the solutions involve $e^{\lambda t}$, so the real parts of the eigenvalues decide growth or decay and the imaginary parts decide the oscillation frequency.