NOTATION · CHEAT SHEET

Eigenvalue Notation at a Glance

The marks from the notation cards across the eigenvalue pages: how each is read and what it means. Every group links to the full card on its page.

CHARACTERISTIC EQUATION

$A\mathbf{v} = \lambda\mathbf{v}$	**A V EQUALS LAMBDA V** λ is the eigenvalue, $\mathbf{v} \neq \mathbf{0}$ the eigenvector.
$\det(A - \lambda I) = 0$	**THE CHARACTERISTIC EQUATION** The I is needed: a number cannot be taken from a matrix.
E_λ	**THE EIGENSPACE FOR LAMBDA** $\text{Null}(A - \lambda I)$: the eigenvectors plus $\mathbf{0}$.

DIAGONALIZATION

$A = PDP^{-1}$	**P D P INVERSE** Eigenvectors as the columns of P, eigenvalues on the diagonal of D.
$P_{:,j} \leftrightarrow D_{jj}$	**MATCHING ORDER** Column j of P goes with entry j of D.
defective	**TOO FEW EIGENVECTORS** Cannot be diagonalised.

PROPERTIES

$f(A)\mathbf{v} = f(\lambda)\mathbf{v}$	**SAME EIGENVECTOR, NEW EIGENVALUE** Powers, inverses and polynomials of A keep $\mathbf{v}$.
$q(A) = c_0 I + \cdots$	**A POLYNOMIAL EVALUATED AT A** The constant term becomes $c_0 I$.
$\mathbf{w}^T A = \lambda \mathbf{w}^T$	**A LEFT EIGENVECTOR** Multiplies from the left; plain eigenvector means the right one.

COMPLEX EIGENVALUES

$\lambda = a \pm bi$	**A CONJUGATE PAIR** Both values occur together for a real matrix.
$A\bar{\mathbf{v}} = \bar{\lambda}\bar{\mathbf{v}}$	**CONJUGATE EIGENPAIR** Conjugating both gives another eigenpair.
$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$	**THE REAL CANONICAL BLOCK** Stands for the pair $a \pm bi$ with real entries.

SAME MARKS, DIFFERENT JOB

$\pm$	Both values together for eigenvalues; elsewhere, a choice between two.
λ vs λI	A number against the matrix it becomes inside $A - \lambda I$.
$\bar{\mathbf{v}}$	Every entry conjugated, not a mean.