WHAT THE SIZE OF THE DETERMINANT SAYS

For an $n \times n$ matrix A, the map $x \mapsto Ax$ scales every n-dimensional volume by the same factor $|\det(A)|$. Comparing that factor with 1 and with 0 gives four regimes.

VOLUME SCALING

$$\text{vol}(A(S)) = |\det(A)| \cdot \text{vol}(S)$$

split by **the size of** $|\det(A)|$

↑

CASE 1

$|\det(A)| > 1$

Expands volumes

every region comes out larger than it went in

↓

CASE 2

$0 < |\det(A)| < 1$

Compresses volumes

every region comes out smaller

=

CASE 3

$|\det(A)| = 1$

Volume-preserving

volumes are unchanged

E.G. rotations and reflections

0

CASE 4

$\det(A) = 0$

Collapses a dimension

the image is a subspace of dimension less than n, and all n-dimensional volumes map to zero

E.G. a hyperplane, a line or a point

✓

TAKEAWAY

The absolute value discards the sign: a rotation and a reflection both have $|\det(A)| = 1$. The sign itself, orientation, is read separately.