**WHAT THE SIGN SAYS**

The columns $\mathbf{u}$ and $\mathbf{v}$ of a $2 \times 2$ matrix span a parallelogram, and $\det(A) = ad - bc$ is its signed area. The sign encodes orientation.

SIGNED AREA

$$\det(A) = ad - bc$$

split by **the sign of** $\det(A)$

**+**

CASE 1

$\det(A) > 0$

### Counterclockwise

rotating from $\mathbf{u}$ toward $\mathbf{v}$ sweeps counterclockwise

E.G. columns $(3, 0)$, $(1, 4)$: $\det = 12$

**−**

CASE 2

$\det(A) < 0$

### Clockwise

the pair is clockwise-ordered

E.G. columns $(1, 4)$, $(3, 0)$: $\det = -12$

**0**

CASE 3

$\det(A) = 0$

### Parallel

the parallelogram degenerates to a line segment with zero area

**TAKEAWAY**

The size is the area, the sign is the orientation: $\det = 12$ and $\det = -12$ give the same area, and swapping the columns reverses the orientation.