

### ① WHAT THE SIGN SAYS

For three vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  in  $\mathbb{R}^3$ , the determinant of the matrix with those columns is the signed volume of the parallelepiped they span. Its sign reflects handedness.

#### SIGNED VOLUME

$$\det(A) = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$$

split by the sign of  $\det(A)$



#### CASE 1

$$\det(A) > 0$$

**Right-handed**

the same orientation as the standard basis  $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$

**E.G.** columns  $(1, 0, 0)$ ,  $(0, 2, 0)$ ,  $(0, 0, 3)$ :  $\det = 6$



#### CASE 2

$$\det(A) < 0$$

**Left-handed**

the opposite orientation; the volume is  $|\det(A)|$

**E.G.** the same columns with the first two swapped:  $\det = -6$



#### CASE 3

$$\det(A) = 0$$

**Flat**

the three vectors are coplanar and the parallelepiped has no volume



#### TAKEAWAY

Swapping two columns flips the sign but not the volume: the box with  $\det = 6$  becomes  $\det = -6$ , left-handed instead of right-handed.