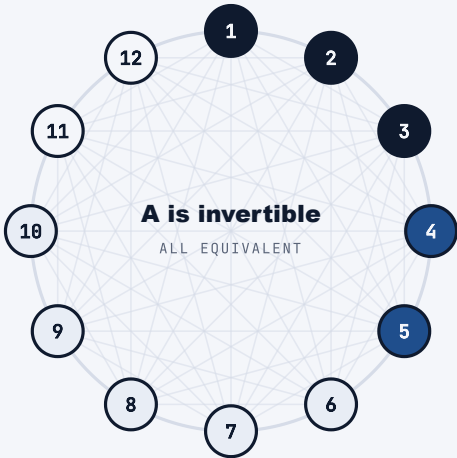


One question, twelve answers

The determinant is one entry in this ring, not its centre. Every statement below implies every other, so any of them could have been the definition and the determinant chosen for convenience rather than priority.



1	$\det(A) \neq 0$	DET
2	A is a product of elementary matrices	DET
3	All eigenvalues of A are nonzero	DET
4	$\text{rank}(A) = n$	RANK
5	The reduced row echelon form of A is I	RANK
6	The columns of A are linearly independent	SPACE
7	The rows of A are linearly independent	SPACE
8	The columns of A span $\mathbb{R}^n$	SPACE
9	The columns of A form a basis for $\mathbb{R}^n$	SPACE
10	The null space of A is $\{\mathbf{0}\}$	SPACE
11	$A\mathbf{x} = \mathbf{b}$ has exactly one solution for every $\mathbf{b}$	SYSTEM
12	$A\mathbf{x} = \mathbf{0}$ has only the trivial solution	SYSTEM

WHAT THE TAGGING EXPOSES

Only three of the twelve mention the determinant at all. The other nine settle invertibility through rank, subspace structure, or solvability — none of which requires computing a determinant. That is worth holding onto, because for large n the determinant is among the **most expensive routes** to an answer the row reduction already gave you on the way.

■ det ×3 ■ rank ×2 □ space ×5 □ system ×2