WORKED EXAMPLE: DIFFERENTIATION

Differentiation as a matrix

With bases fixed, even differentiation becomes a matrix, and calculus on polynomials turns into arithmetic on coordinates.

BEFORE

The map

$$T(a + bx + cx^2) = b + 2cx$$

Differentiation from P_2 to P_1

FIX BASES, READ COLUMNS

bases {1, x, x²} and {1, x}, one column per basis polynomial

▼ WHY 2 × 3

AFTER

The matrix

$$[T] = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Columns: T(1), T(x), T(x²) in coordinates

WHAT CHANGED

A rule on polynomials became a 2 × 3 array: coordinates (a, b, c) go in, (b, 2c) come out.

WHAT STAYED

The map: multiplying coordinates by [T] differentiates.