CONTRADICTIONS

Reducing to X ∧ ¬X

Reach X ∧ ¬X through equivalences alone, and the contradiction is proved without writing a single row.

BEFORE

Disjunction with both parts denied

$$(P \vee Q) \wedge \neg P \wedge \neg Q$$

At least one of P or Q, yet P false and Q false

DE MORGAN'S LAW

¬P ∧ ¬Q ≡ ¬(P ∨ Q)

FLIP THE CONNECTIVE

AFTER

Pattern X ∧ ¬X

$$(P \vee Q) \wedge \neg(P \vee Q)$$

With X = P ∨ Q: the basic contradiction pattern

WHAT CHANGED

The two separate denials ¬P and ¬Q merged into one negation of the whole disjunction.

WHAT STAYED

The truth value in every row: each step is an equivalence.