BEFORE YOU READ

Ten templates for counting problems

Almost every counting problem matches one of ten standard scenarios. The first split is whether the order of the chosen items matters.

- L0 **Standard counting scenarios** ten templates
  - L1 **Permutation scenarios** order matters
    - L2 **Full permutation** arrange n distinct items in a row — $n!$
    - L2 **With identical items** n items, n_1 ... n_k of each type — $n!/(n_1!...n_k!)$
    - L2 **Partial, no repetition** arrange r chosen from n — $n!/(n-r)!$
    - L2 **With repetition** fill r positions from n choices — n^r
    - L2 **Circular** arrange n items in a circle — $(n-1)!$
  - L1 **Combination scenarios** order does not matter
    - L2 **Simple combination** choose r items from n — $C(n, r)$
    - L2 **Partition into groups** split n items into labeled groups — multinomial
    - L2 **Weak composition** n identical items into r bins, empty allowed — $C(n+r-1, r-1)$
    - L2 **Strong composition** n identical items into r bins, none empty — $C(n-1, r-1)$
    - L2 **Distribution into cells** n distinct items into r labeled cells — r^n

TAKEAWAY

Find the template, then the formula

Decide whether order matters, then whether items repeat or are identical; the matching scenario hands you the formula.