A TEST WITHOUT EVALUATING

The conjugate classifies a complex number $z = a + bi$: compare $\bar{z}$ with z. The two existence theorems give the first two outcomes.

EXISTENCE THEOREMS

$\bar{z}$ compared with z

split by **what conjugation does to z**

CASE 1

$\bar{z} = z$

Real

conjugation leaves it fixed: $b = 0$, it lives on the real axis

CASE 2

$\bar{z} = -z$

Pure imaginary

conjugation negates it: $a = 0$, it lives on the imaginary axis; see **pure imaginary numbers**

CASE 3

neither

Off both axes

nonzero real and imaginary parts: the interior of the complex plane

TAKEAWAY

To check whether an expression yields a real result, simply check whether it equals its own conjugate.