COMPLEX NUMBERS

What each operation does to conjugate, modulus and argument

The conjugate passes through everything; the modulus and the argument each have their own gaps.

✓ EXACT LAW · ● BOUND ONLY · ◐ UP TO FULL TURNS · ✕ NO SIMPLE RULE

OPERATION	Conjugate	Modulus	Argument
Sum $z_1 + z_2$	✓ EXACT LAW $\bar{z}_1 + \bar{z}_2$	● BOUND ONLY $\lvert z_1 + z_2\rvert \le \lvert z_1\rvert + \lvert z_2\rvert$	✕ NO SIMPLE RULE
Difference $z_1 - z_2$	✓ EXACT LAW $\bar{z}_1 - \bar{z}_2$	● BOUND ONLY $\lvert z_1 - z_2\rvert \ge \big\lvert\lvert z_1\rvert - \lvert z_2\rvert\big\rvert$	✕ NO SIMPLE RULE
Product $z_1 z_2$	✓ EXACT LAW $\bar{z}_1\,\bar{z}_2$	✓ EXACT LAW $\lvert z_1\rvert\,\lvert z_2\rvert$	◐ UP TO FULL TURNS $\arg z_1 + \arg z_2$
Quotient z_1/z_2	✓ EXACT LAW $\bar{z}_1/\bar{z}_2$	✓ EXACT LAW $\lvert z_1\rvert/\lvert z_2\rvert$	◐ UP TO FULL TURNS $\arg z_1 - \arg z_2$
Power z^n	✓ EXACT LAW $(\bar{z})^n$	✓ EXACT LAW $\lvert z\rvert^n$	◐ UP TO FULL TURNS $n \arg z$ (De Moivre)
Conjugation $\bar{z}$	✓ EXACT LAW z (conjugating twice)	✓ EXACT LAW $\lvert z\rvert$, unchanged	◐ UP TO FULL TURNS $-\arg z$

Argument laws hold modulo 2π; with the principal value Arg, a result may need 2π added or subtracted to stay in $(-\pi, \pi]$.