NOTATION · CHEAT SHEET

Complex Number Notation at a Glance

The marks from the notation cards across the complex-number pages: how each is read and what it means. Every group links to the full card on its page.

BASICS

$z,\ w$	**Z, W: COMPLEX NUMBERS** The usual letters for complex numbers, as x and y are for reals.
$a + bi$	**A PLUS B I** Real part a, imaginary part b; both are real numbers.
$\mathbb{C}$	**C, THE COMPLEX NUMBERS** The set of all $a + bi$; it contains $\mathbb{R}$.

IMAGINARY NUMBERS

i	**I, THE IMAGINARY UNIT** The number with $i^2 = -1$.
$\sqrt{-1}$	**THE SQUARE ROOT OF MINUS ONE** Another name for i; root rules for reals fail with it.
bi	**B I** A pure imaginary number: real part 0.

ALGEBRAIC FORM

$\mathrm{Re}(z),\ \mathrm{Im}(z)$	**REAL PART, IMAGINARY PART** For $z = a + bi$: a and b. $\mathrm{Im}(z)$ is b, not bi.
$a + (-b)i$	**A PLUS NEGATIVE B, I** Written $a - bi$; the imaginary part is $-b$.
$z = x + yi$	**Z EQUALS X PLUS Y I** Same form with plane letters: the point (x, y).

COMPLEX CONJUGATE

$\bar{z}$	**Z BAR, THE CONJUGATE** For $z = a + bi$, $\bar{z} = a - bi$: the mirror image across the real axis.
$\overline{z_1 + z_2}$	**THE CONJUGATE OF THE SUM** The bar covers the whole expression; it equals $\bar{z}_1 + \bar{z}_2$.
z^*	**Z STAR** The same conjugate, as written in physics and engineering.

ABSOLUTE VALUE

$\lvert z \rvert$	**THE MODULUS OF Z** Distance from 0: $\sqrt{a^2 + b^2}$.
$\lvert z \rvert^2 = z\bar{z}$	**MOD Z SQUARED** Equals $a^2 + b^2$, with no root to take.
$\lvert z - c \rvert = R$	**MOD Z MINUS C EQUALS R** The circle of radius R around c.

ADDITIVE INVERSE

$-z$	**NEGATIVE Z** Both parts change sign: $-a - bi$.
$-(a + bi)$	**MINUS, QUANTITY A PLUS B I** The brackets carry the minus to both parts.
$z - w$	**Z MINUS W** Adding the inverse: $z + (-w)$.

MULTIPLICATIVE INVERSE

$z^{-1},\ \frac{1}{z}$	**Z INVERSE; ONE OVER Z** The number with $z \cdot z^{-1} = 1$; it equals $\frac{\bar{z}}{\lvert z \rvert^2}$.
$(3 + 2i)^{-1}$	**THE INVERSE OF THREE PLUS TWO I** Found by multiplying top and bottom by the conjugate: $\frac{3-2i}{13}$.

TRIGONOMETRIC FORM

$r,\ \theta$	**R AND THETA, THE POLAR PAIR** Distance from 0 and angle from the positive real axis.
$\arg(z)$	**THE ARGUMENT OF Z** Any angle of z; there are infinitely many, 2π apart.
$\mathrm{Arg}(z)$	**THE PRINCIPAL ARGUMENT** One chosen angle, usually in $(-\pi, \pi]$; some texts use $[0, 2\pi)$.
$\mathrm{cis}\,\theta$	**SIS THETA** Short for $\cos\theta + i\sin\theta$.

EXPONENTIAL FORM

$e^{i\theta}$	**E TO THE I THETA** Equals $\mathrm{cis}\,\theta$ (Euler's formula): a point on the unit circle.
$re^{i\theta}$	**R E TO THE I THETA** The whole number z: length r, angle θ.
$e^{i(\theta+2\pi k)}$	**THETA PLUS TWO PI K** The same number for every integer k.

DE MOIVRE'S THEOREM

z_k	**Z SUB K, THE K-TH ROOT** The roots of $z^n = w$, numbered $k = 0, \ldots, n-1$.
ω	**OMEGA** A primitive root of unity: its powers give all n roots of 1.
$R^{1/n}$	**R TO THE ONE OVER N** The real root of the length; the complex root $w^{1/n}$ has n values.

SAME MARKS, DIFFERENT JOB

$\lvert z \rvert$	The modulus here; for a real number, the absolute value; for a matrix, the determinant.
$\bar{z}$	The conjugate here; in statistics a bar means a mean, in sets a complement.
z^{-1} vs z^*	The reciprocal, against the conjugate: they agree only on the unit circle.
$w^{1/n}$	n values for a complex w; a single real root for a positive real number.