THE MODULUS INVERTS

The inverse of z has modulus $\frac{1}{|z|}$: a number far from the origin has an inverse close to it, and vice versa. Comparing $|z|$ with 1 decides which.

MULTIPLICATIVE INVERSE

$$|z^{-1}| = \frac{1}{|z|}$$

split by $|z|$ compared with 1

CASE 1

$|z| > 1$

Moves closer

the inverse has modulus less than one

E.G. $|z| = 5 \Rightarrow |z^{-1}| = \frac{1}{5}$

CASE 2

$|z| < 1$

Moves farther out

the inverse has modulus greater than one

E.G. $|z| = \frac{1}{3} \Rightarrow |z^{-1}| = 3$

CASE 3

$|z| = 1$

Stays on the unit circle

the same distance from the origin after inversion

THE ARGUMENT NEGATES

In every case $\arg(z^{-1}) = -\arg(z)$: the inverse reflects z across the real axis and scales its distance by $\frac{1}{|z|^2}$.